NON-LEPTONIC DECAYS OF BEAUTY HADRONS – FROM PHENOMENOLOGY TO THEORY [1]

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ABSTRACT

In the last few years considerable progress has been achieved in our understanding of the decays of heavy flavour hadrons. One can now calculate *inclusive* transition rates in QCD proper through an expansion in inverse powers of the heavy flavour quark mass without recourse to phenomenological assumptions. The non-perturbative contributions are treated systematically in this way; they are found to produce corrections of order a few percent in beauty decays, i.e. typically somewhat smaller than the perturbative corrections. One finds, among other things: (a) The lifetime of B^- mesons is predicted to be longer than that of B^0 mesons by several percent. (b) The QCD prediction for the semileptonic branching ratio of B mesons appears to exceed present experimental values. We discuss the implications of this discrepancy. The phenomenological engineering that has been developed for the description of *exclusive* two-body modes of B mesons has reached a mature stage and awaits more precise and detailed experimental tests. First steps towards a genuine QCD treatment of these modes are being made.

'Anyone who keeps the ability to see beauty never grows old'   Franz Kafka

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1 Introduction

In the two years since the appearance of the first edition of this book our understanding of the decays of beauty hadrons has been improved quite significantly. The availability of better and more comprehensive data has obviously helped here – but so has the emergence of more reliable theoretical tools! Rather than having to rely on phenomenological models we can now employ approaches that are directly based on QCD with no appeal to a 'deus ex machina', at least for semileptonic and inclusive non-leptonic decays. Therefore the article on non-leptonic beauty decays had to be re-written completely. *Inclusive* decays of beauty hadrons can now be treated in QCD in a quantitative fashion with systematic estimates of the uncertainties.

The employed formalism combines the heavy quark mass expansion with other elements derived from QCD proper. Its technical foundation is the Wilson operator product expansion (OPE) [1]. The idea of applying OPE to the inclusive heavy flavour decays had emerged in in the eighties [2] and has now grown into a well-developed scheme [3, 4, 5, 6].

The basic procedure can be illustrated by a simple analogy with nuclear β decay. There are two effects distinguishing leptons in the decay of a neutron bound inside a nucleus from those in a decay of a free neutron:

(a) nuclear binding effects;

(b) Pauli statistics correlating the electrons surrounding the nucleus with those emerging from the β decay.

The typical energies of the bound electrons ϵ_{el} are certainly small compared to E_{rel}, the energy released in the decay; let us assume – although this is not true in reality – that also the nuclear binding energies ϵ_{nucl} were small compared to E_{rel}. In that case nuclear β decays would proceed like the decays of *free* neutrons to a good approximation; corrections to this simple 'spectator' picture could be computed via an expansion in powers of ϵ_{nucl}/E_{rel}, ϵ_{el}/E_{rel} In practice, however, the corrections for nuclear β decay are incorporated by explicitly using the wavefunctions of the bound nucleons and electrons.

There arise analogous corrections to the decay rate for a quark Q inside a hadron:

(a) interactions of the decaying quark with other partons in the hadron[1].

(b) Pauli interference effects of the decay products with other partons in the hadron; e.g.: $b\bar{u} \to c\bar{u}d\bar{u}$.

The difference with the example above is quite obvious: even in the limit $m_Q \to \infty$ a non-relativistic bound state treatment is inapplicable since the dynamical degrees of freedom of the heavy flavour hadron H_Q cannot be fully described by a hadronic wavefunction! We will return to this point later on. The most reliable approach is then to evaluate weak decay rates of heavy flavour hadrons through an expansion in powers of μ_{had}/m_Q; m_Q – the heavy flavour quark mass – is a measure of the energy release in the decay and μ_{had} represents ordinary hadronic scales which enter through the matrix elements for heavy flavour hadrons. Since $\mu_{had} < 1$ GeV (details will be given below) one expects such an expansion to work quite well for beauty decays.

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[1]There is also the annihilation of the heavy quark with the light (spectator) antiquark for which an analogy is found in the K capture of bound electrons by a heavy nucleus.

The example given above illustrates two important features of our analysis: it applies to *inclusive* non-leptonic and semileptonic transitions, and the usefulness of such an expansion rests on a large *energy release* in the decay; i.e. in $Q_1 \to Q_2$'W', where 'W' denotes $q'\bar{q}$ or $l\nu$ one requires $m_{Q_1} - m_{Q_2} \gg \mu_{had}$. This is in contrast to the approach usually referred to as the heavy quark symmetries [7] which applies to *exclusive* semileptonic amplitudes provided that the initial and the final quarks are both heavy, i.e. m_{Q_1}, $m_{Q_2} \gg \mu_{had}$ but the ratio m_{Q_2}/m_{Q_1} is otherwise arbitrary. The QCD-based description of the inclusive decays of heavy flavours naturally incorporates many elements of Heavy Quark Effective Theory (*HQET*) [8, 9]. The systematic expansion in m_Q^{-1} is inherent to both, and it allows us – as we will see shortly – to determine the size of important input parameters for our analysis.

This survey will be organized as follows: in Sect. 2 we review the general features of the $1/m_Q$ expansion for inclusive decay rates; in Sect. 3 we present quantitative predictions for lifetime ratios, semileptonic branching ratios and radiative widths and discuss their theoretical uncertainties; in Sect. 4 we address exclusive non-leptonic decays before giving a summary and an outlook in Sect. 5.

2   Inclusive Decay Rates of Beauty Hadrons: Formalism

As mentioned above, the systematic approaches to inclusive heavy flavor decays date back to the beginning of the eighties. Unitarity relates inclusive decay rates to the imaginary part of certain forward 'scattering' amplitudes; this is a rather trivial observation [10], yet it opened the way for the consistent use of OPE. The starting object of our analysis is the transition operator $\hat{T}(b \to f \to b)$ describing the forward scattering of b quarks via an intermediate state f. To second order in the weak interactions the transition operator is given by [10]

$$\hat{T}(b \to f \to b) = i \int d^4x \{\mathcal{L}_W(x)\mathcal{L}_W^\dagger(0)\}_T, \tag{1}$$

where $\mathcal{L}_W$ denotes the relevant effective weak Lagrangian and $\{.\}_T$ is the time-ordered product. Treating m_b as a large parameter a Wilson OPE allows expressing the non-local operator $\hat{T}$ as an infinite sum of local operators of increasing dimension with coefficients containing higher and higher powers of $1/m_b$. The lowest dimensional term will dominate in the limit $m_b \to \infty$; for beauty decays that is the dimension three operator $\bar{b}b$. The width for the decay of a beauty hadron H_b into an inclusive final state f is obtained by taking the expectation value of $\hat{T}$ between the state H_b. Through order $1/m_b^3$ one finds [2, 4, 5, 11]:

$$\Gamma(H_b \to f) = \frac{G_F^2 m_b^5}{192\pi^3}|KM|^2 \left[c_3(f)\frac{\langle H_b|\bar{b}b|H_b\rangle}{2M_{H_b}} + \frac{c_5(f)}{m_b^2}\frac{\langle H_b|\bar{b}i\sigma_{\mu\nu}G_{\mu\nu}b|H_b\rangle}{2M_{H_b}} + \right.$$

$$\left. + \sum_i \frac{c_6^{(i)}(f)}{m_b^3}\frac{\langle H_b|(\bar{b}\Gamma_i q)(\bar{q}\Gamma_i b)|H_b\rangle}{2M_{H_b}} + \mathcal{O}(1/m_b^4) \right], \tag{2}$$

where the dimensionless coefficients $c_i(f)$ depend on the parton level characteristics of f (such as the ratios of the final state quark masses to m_b), KM denotes the

appropriate combination of weak mixing angles and $G_{\mu\nu}$ the gluonic field strength tensor. The last term implies also the summation over the four-fermion operators with different light flavours q. Notice that the factor $1/2M_{H_b}$ reflects the relativistic normalization of the state $|H_b\rangle$.

It is through the expectation values of local operators appearing on the right-hand side of eq. (2) that the dependence on the decaying *hadron*, and on non-perturbative forces in general, enters instead of entering through the wavefunctions as in nuclear β decay. Since these are matrix elements for real b hadrons one sees that $\Gamma(H_b \to f)$ is indeed expanded into a power series in μ_{had}/m_b. The heavy quark expansion enables us to express decay rates in terms of the expectation values of local operators taken between beauty hadrons. Using heavy quark expansions one can relate some of these hadronic expectation values – and in particular those that appear in the leading terms of the expansion in eq. (2) – to other observables and thus extract their size, as we will discuss now.

(i) To determine $\langle H_b|\bar{b}b|H_b\rangle$ one expresses, *via* equations of motion, the operator $\bar{b}b$ through another series in inverse powers of m_b (which constitutes a static expansion) [5]:

$$\langle H_b|\bar{b}b|H_b\rangle =$$

$$\langle H_b|v_\mu\bar{b}\gamma_\mu b - \frac{1}{2m_b^2}\bar{b}[(iv \cdot D)^2 - (iD)^2]b + \frac{1}{4m_b^2}\bar{b}i\sigma \cdot Gb|H_b\rangle + \mathcal{O}(1/m_b^4) \qquad (3)$$

with v_μ denoting the four-velocity of the heavy hadron. The first operator appearing on the *RHS* of eq. (3) is actually the Noether current for the (global) heavy flavour quantum number; its expectation value is thus determined by the beauty content of H_b and therefore

$$\langle H_b|\bar{b}b|H_b\rangle/(2M_{H_b}) = 1 + \mathcal{O}(1/m_b^2). \qquad (4)$$

It is this term that exactly reproduces the parton model spectator result in the limit $m_b \to \infty$, which attributes equal lifetimes to all hadrons of a given heavy flavour.

(ii) The chromomagnetic operator $\bar{b}i\sigma \cdot Gb$ appears directly in eq. (2) and indirectly through the expansion of $\bar{b}b$. Its expectation value vanishes for the baryon Λ_b:

$$\langle \Lambda_b|\bar{b}i\sigma \cdot Gb|\Lambda_b\rangle \simeq 0 \qquad (5)$$

For the B meson it is given by the hyperfine splitting of the B^* and B masses:

$$\langle B|\bar{b}i\sigma \cdot Gb|B\rangle/(2M_{H_b}) \simeq \frac{3}{2}(M_{B^*}^2 - M_B^2) \simeq 0.74 \text{ GeV }^2 \qquad (6)$$

(iii) The second operator on the *RHS* of eq. (3) describes the kinetic energy of the b quark moving under the influence of the non-trivial gluon background field prevalent inside the hadron H_b: $\langle H_b|\bar{b}[(iv \cdot D)^2 - (iD)^2]b|H_b\rangle \simeq \langle H_b|\bar{b}(i\vec{D})^2b|H_b\rangle \equiv \langle(\vec{p}_b)^2\rangle_{H_b} \cdot (2M_{H_b})$. Its appearance in eq.(3) has a very transparent meaning:

$$\frac{\langle H_b|\bar{b}b|H_b\rangle}{2M_{H_b}} = 1 - \frac{\langle(\vec{p}_b)^2\rangle_{H_b}}{2m_b^2} + \frac{3}{8}\frac{M_{B^*}^2 - M_B^2}{m_b^2} + \mathcal{O}(1/m_b^3) \qquad (7)$$

The first two terms on the *RHS* of eq. (7) represent the mean value of the factor $\sqrt{1 - \vec{v}^2}$ due to time dilation slowing down the decay of the b quark in a moving

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frame. The heavy quark expansion relates the *difference* in the expectation values of the kinetic energy operator for heavy flavour baryons and mesons to the masses of the charm and beauty baryons and mesons [12]:

$$\langle (\vec{p}_b)^2 \rangle_{\Lambda_b} - \langle (\vec{p}_b)^2 \rangle_B \simeq \frac{2m_b m_c}{m_b - m_c} \cdot \{ [\langle M_B \rangle - M_{\Lambda_b}] - [\langle M_D \rangle - M_{\Lambda_c}] \} \qquad (8)$$

where $\langle M_{B,D} \rangle$ denote the 'spin averaged' meson masses,

$$\langle M_B \rangle \equiv \frac{1}{4}(M_B + 3M_{B^*}) \qquad (9)$$

and likewise for $\langle M_D \rangle$. Eq. (9) implies that c quark can be also treated as heavy so that $\langle (\vec{p}_b)^2 \rangle_{H_b} = \langle (\vec{p}_c)^2 \rangle_{H_c}$. From the present data we obtain

$$\langle (\vec{p}_b)^2 \rangle_{\Lambda_b} - \langle (\vec{p}_b)^2 \rangle_B \simeq (-0.03 \pm 0.17) \text{ GeV}^2 \qquad (10)$$

where the error is due to the ± 30 MeV experimental uncertainty in M_{Λ_b}. The Λ_b mass has to be measured with better than a 10 MeV precision to make this relation numerically useful.

Using the commutator algebra of the covariant derivatives iD_μ one can derive an 'uncertainty principle' for their components and thus establish a model independent lower bound [13]:

$$\langle (\vec{p}_b)^2 \rangle_B \geq 0.18 \text{ (GeV)}^2. \qquad (11)$$

An existing analysis based on QCD sum rules yields a value only three times larger than this lower limit [14]:

$$\langle (\vec{p}_b)^2 \rangle_B \simeq 0.6 \text{ (GeV)}^2. \qquad (12)$$

(iv) The expectation value for the four-quark operators looks very similar to the one controlling $B^0 - \bar{B}^0$ oscillations:

$$\langle B(p)|(\bar{b}_L \gamma_\mu q_L)(\bar{q}_L \gamma_\nu b_L)|B(p) \rangle \simeq f_B^2 p_\mu p_\nu \qquad (13)$$

where we have set the so-called bag factor to unity.

Before discussing the phenomenology that is obtained from eq.(2) we want to point out seven basic observations:

(1) The most important aspect – quantitatively as well as conceptually – of the expression in eq. (2) is contained in the element that is missing there: there are no non-perturbative contributions of order $1/m_b$ to fully integrated rates [5]! The numerical impact of this fact is obvious: since the leading non-perturbative corrections then arise on the $1/m_b^2$ level, they fade away quickly with increasing heavy flavour quark mass and amount, for the beauty decays, to effects of order of several percent only since the scale is set by the quantities

$$G_B \equiv \frac{\langle B|\bar{b} i\sigma \cdot Gb|B \rangle}{2m_b^2 \cdot (2M_{H_b})} \simeq 0.015, \qquad (14a)$$

$$K_B \equiv \frac{\langle \vec{p}_b^2 \rangle_B}{m_b^2} \sim 0.015. \qquad (14b)$$

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Let us note in passing that the analogous contributions in charm decays are much larger since they are amplified by a factor $(m_b/m_c)^2 \sim 10$!

The conceptual relevance of the absence of $1/m_b$ terms is of a more subtle, but not less important nature. On the one hand it confirms the conjecture that the (current) quark mass m_b rather than the hadron mass M_B represents the natural expansion parameter. For if the total width were correctly expressed in terms of M_B then a term linear in $1/m_b$ had to appear: $\Gamma(B) \propto G_F^2 M_B^5 = G_F^2 m_b^5 (1 + 5\bar{\Lambda}_B/m_b + ...)$ with the notation $M_B = m_b + \bar{\Lambda}_B + \mathcal{O}(1/m_b)$. This is actually more than an academic point for it would have a significant impact on the lifetime difference between B mesons and Λ_b baryons: $(\tau(\Lambda_b) - \tau(B))/\tau(B) \propto 1/m_b$ since $\bar{\Lambda}_{\Lambda_b} \neq \bar{\Lambda}_B$! On the other hand it is quite instructive to understand the dynamical reason why no linear $1/m_b$ terms arise in total decay rates whereas they do appear in mass formulae. Looking at the explicit QCD calculation given in ref. [5] one can already infer why no contribution of order $1/m_b$ arises: only a dimension four operator could generate such a term and there simply does not exist such an appropriate operator that is gauge invariant and cannot be absorbed into the dimension three operator $\bar{b}b$ by means of equations of motion. (This means that proper care has to be applied in employing a consistent definition of the heavy flavour mass m_b.) It is therefore the colour symmetry, i.e. the fact that the colour flow is conserved that ensures the absence of a $1/m_b$ correction. This correction is absent also for differential distributions like the lepton spectra in semileptonic decays *outside* the end-point region. The size of this end-point domain is of order μ_{hadr}/m_b if all energies are measured in units of m_b. In this domain we cannot limit ourselves to the operators of the lowest dimension, and one needs to sum up an infinite series to determine the shape of the distribution which leads to modification of the spectrum of order unity in the end-point region. It is remarkable that the full integral over the end-point region still has no corrections proportional to $1/m_b$!

(2) The distinction between beauty baryon and meson decays is systematically expressed through differences in the appropriate expectation values of the same operators, see eq.(2). Differences arise first in the leading non-perturbative corrections of order $1/m_b^2$, see eqs.(5, 6, 8). Yet apart from some small $SU(3)_{FL}$ breaking effects they affect B_d, B^- and B_s decays in the same way; likewise for Λ_c vs. D^0, D^+ and D_s decays. Numerically they are comparable to – actually typically smaller than – *perturbative* corrections in beauty decays whereas they dominate perturbative effects in charm decays.

(3) Local four-quark operators of dimension six finally produce differences between all the B meson lifetimes:
$$\frac{\Gamma_{nonspect}(B)}{\Gamma_{spect}(B)} \propto \frac{f_B^2}{m_b^2}$$

which formally scales like $1/m_b^3$. These effects are therefore predicted to be greatly reduced relative to the considerable lifetime differences in the D system.

(4) Contributions of order $1/m_b^4$ are generated by dimension seven operators. Yet it appears to be practically unfeasible to determine the expectation values for all or even most of them. What seems possible – although it has not been done yet – is to analyze a small subset of them, namely those yielding *factorizable* contributions, for

a more detailed error estimate.

(5) It is intuitively obvious that the b quark does not rest inside a beauty hadron H_b, but will move around with a certain 'Fermi momentum'. This has been incorporated into phenomenological models of inclusive heavy flavour decays, first in ref. [16]. It has been first stated in ref. [6] and then further discussed in refs. [17, 13, 33, 34] that this notion of Fermi motion of a heavy flavour quark finds a natural home also in a rigorous QCD treatment; yet, strictly speaking, no hadronic wavefunction for H_b in the usual sense can be found that reproduces the Fermi motion beyond the second moment $\langle (\vec{p})^2 \rangle$. We will not discuss this in any detail here since $\langle (\vec{p})^2 \rangle$ has a reduced numerical relevance in fully integrated rates (although it is of crucial importance for shaping the end-point spectrum in the inclusive semileptonic and radiative decays).

(6) While our expressions for inclusive decay rates are firmly based on QCD, one has to invoke explicitly – and at the moment additionally – the concept of duality. The operator product expansion is unambigously defined in the Euclidean domain, yet the kinematics of the actual decay are of a time-like Minkowskian nature. It is then conceivable – although it has never been illustrated in a clear way – that a translation between the Euclidean and the Minkowskian expression that is based on *local* duality does not hold in non-leptonic or for that matter even in semileptonic decays. We view this as a mathematical rather than as a physical caveat. The conjecture of duality can be supported by some general arguments, yet their discussion would lead beyond the scope of this article.

(7) The expression in eq. (2) is based on OPE where one separates short distance and long distance dynamics by isolating the latter in the local operators and their matrix elements and letting the former determine the c number coefficients $c_i(f)$. In actual calculations one goes one step further: one computes the coefficients $c_i(f)$ in perturbation theory alone although non-perturbative short distance contributions do exist. The latter are guestimated (and in τ decays found [18]) to be quite small. We adopt this procedure which we refer to as the 'Standard Version' of OPE although we will also comment on possible limitations later on.

3 Phenomenology of Inclusive Beauty Decays

There are five types of inclusive observables we will discuss here, namely: (1) total lifetimes; (2) semileptonic branching ratios; (3) other inclusive non-leptonic decays; (4) radiative decays; (5) charm multiplicity in the final state and (6) $B \to$ charmonia + X.

3.1 Total Lifetimes

There exists a triple motivation behind measuring the lifetimes of different species of beauty hadrons as precisely as possible: (a) Representing the most inclusive quantity it provides a clear and well-defined challenge to theory. (b) It allows to obtain the semileptonic *width* from the measured semileptonic branching ratios; from this width one extracts the KM parameter $|V_{cb}|$ etc. (c) It is a pre-requisite of a detailed analysis of $B^0 - \bar{B}^0$ oscillations.

There is no basic uncertainty about the weak forces driving non-leptonic beauty decays: at the scale M_W they are given by the Lagrangian

$$\mathcal{L}_W^{\Delta B=1}(\mu = M_W) = \frac{4G_F}{\sqrt{2}}[V_{cb}\bar{c}_L\gamma_\mu b_L + V_{ub}\bar{u}_L\gamma_\mu b_L] \cdot [V_{ud}^*\bar{d}_L\gamma_\mu u_L + V_{cs}^*\bar{s}_L\gamma_\mu c_L]. \quad (15)$$

where we have ignored Cabibbo suppressed transitions and also the $b \to t$ coupling since we will not discuss $B^0 - \bar{B}^0$ oscillations and Penguin contributions here. Radiative QCD corrections lead to a well-known renormalization at scale m_b, which is often referred to as ultra-violet (UV) renormalization:

$$\mathcal{L}_W^{\Delta B=1}(\mu = m_b) = \frac{4G_F}{\sqrt{2}}V_{cb}V_{ud}^*\{c_1(\bar{c}_L\gamma_\mu b_L)(d_L\gamma_\mu u_L) + c_2(\bar{d}_L\gamma_\mu b_L)(\bar{c}_L\gamma_\mu u_L)\} \quad (16)$$

for $b \to c\bar{u}d$ and likewise for $b \to c\bar{c}s$ etc. transitions; the QCD corrections are lumped together into the coefficients c_1 and c_2 with

$$c_1 = \frac{1}{2}(c_+ + c_-), \quad c_2 = \frac{1}{2}(c_+ - c_-) \quad (17a)$$

$$c_\pm = [\frac{\alpha_S(M_W^2)}{\alpha_S(m_b^2)}]^{\gamma_\pm}, \quad \gamma_+ = \frac{6}{33 - 2N_f} = -\frac{1}{2}\gamma_- \quad (17b)$$

in the leading log approximation. Numerically this amounts to

$$c_1(LL) \simeq 1.1, \quad c_2(LL) \simeq -0.23 \quad (18)$$

Including next-to-leading log corrections one obtains

$$c_1(LL + NLL) \simeq 1.13, \quad c_2(LL + NLL) \simeq -0.29 \quad (19)$$

i.e. a mild enhancement of the original coupling together with the appearence of an induced operator with a different color flow. Later we will also include [19] the so-called 'hybrid' renormalization reflecting radiative corrections in the domain from m_b down to μ_{had} [20].

As already stated in Sect. 2 differences between B *meson* lifetimes arise on the $1/m_b^3$ level generated by local four-quark operators $(\bar{b}_L\gamma_\mu q_L)(\bar{q}_L\gamma_\nu b_L)$. Based on this scaling law one can already infer from the observed D meson lifetime ratios that the various B meson lifetimes will differ by no more than 10 percent or so.

In phenomenological models two distinct mechanisms producing lifetime differences had been noted, namely
  • Weak Annihilation (WA) and
  • Pauli Interference (PI) [21]
in qualitative analogy to the situation in nuclear β decay as explained in the Introduction. In the $1/m_b$ expansion they emerge as follows. There are two types of four-quark operators which are distinguished by how the light quark flavours are connected inside the hadron H_b. This can be seen from Figs. 1. Upon integrating out the c, $\bar{q}$ and q' fields in the diagram of Fig. 1a where the square boxes represent $\mathcal{L}_W^{\Delta B=1}$ one obtains the operator $\bar{b}b$; cutting then the q' line in Fig. 1a and connecting it to

the q' constituent of the B meson, as shown in Fig. 1b, one has a WA transition operator; cutting instead the $\bar{q}$ line and connecting it to the B constituents, see Fig. 1c, leads to the four-fermion operator describing PI.

It turns out that the WA processes can change B lifetimes by no more than, say, 1%; due to interference with the spectator reaction they could actually *prolong* $\tau(B_d)$ relative to $\tau(B^-)$ rather than reduce it [4]! The dominant effect is provided by PI which produces an additional contribution to the B^- width:

$$\Gamma(B^-) = \Gamma_{Spect}(B) + \Delta\Gamma_{PI}(B^-) \tag{20a}$$

$$\Delta\Gamma_{PI}(B^-) \simeq \Gamma_0 \cdot 24\pi^2 \frac{f_B^2}{M_B^2}[c_+^2 - c_-^2 + \frac{1}{N_C}(c_+^2 + c_-^2)], \;\; \Gamma_0 = \frac{G_F^2 m_b^5}{192\pi^3}|V(cb)|^2 \tag{20b}$$

where the appearance of f_B reflects the fact that PI – like WA – requires the spatial overlap of two (anti)quark fields.

Eq. (20b) exhibits an intriguing result: $\Delta\Gamma_{PI}(B^-)$ is positive for $c_+ = 1 = c_-$, i.e. PI acts *constructively*. Radiative QCD corrections with $c_+ \simeq 0.84$, $c_- \simeq 1.42$ turn PI into a *destructive* interference which prolongs $\tau(B^-)$ by a tiny amount. In eq. (20b) only UV renormalization has been applied. Hybrid renormalization amplifies this effect considerably and one obtains [4, 19]:

$$\Delta\Gamma_{PI}(B^-) \simeq \Gamma_0 \cdot 24\pi^2 \frac{f_B^2}{M_B^2}\kappa^{-4}[(c_+^2 - c_-^2)\kappa^{9/2} + \frac{c_+^2 + c_-^2}{3} - \frac{1}{9}(\kappa^{9/2} - 1)(c_+^2 - c_-^2)],$$

$$\kappa \equiv [\frac{\alpha_S(\mu_{had}^2)}{\alpha_S(m_b^2)}]^{1/b}, \; b = 11 - \frac{2}{3}n_F \tag{21}$$

Altogether one finds:

$$\frac{\tau(B^-)}{\tau(B_d)} \simeq 1 + 0.05 \cdot \frac{f_B^2}{(200 \text{ MeV })^2} \;, \tag{22}$$

i.e. the lifetime of a charged B meson is predicted to exceed that of a neutral B meson by several percent, but not more than ten percent [2]. Corrections of order $1/m_b^4$ which have been ignored here are unlikely to change this prediction significantly; this statement will however be qualified below in our discussion of the semileptonic B branching.

One also expects

$$\bar{\tau}(B_d) \simeq \bar{\tau}(B_s) \tag{23}$$

where $\bar{\tau}$ denotes the average lifetime of the two mass eigenstates in the $B^0 - \bar{B}^0$ system. It is at least amusing to note that the largest lifetime difference among B

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[2]It should be noted that an analogous analysis yields $\tau(D^+)/\tau(D^0) \sim 2$, which is quite consistent with the observed value of $\simeq 2.5$. Yet one has to keep in mind that the charm quark mass is not much larger than typical hadronic masses; the convergence of the $1/m_c$ expansion is thus too slow, if it happens at all, to yield a better than semi-quantitative description of non-leptonic charm decays. For a recent review of the theoretical situation here see ref. [22].

mesons is most likely produced by a subtle mechanism, namely B_s-$\bar{B}_s$ oscillations with both PI-like and WA-like mechanisms contributing [23]:

$$\frac{\Delta\Gamma(B_s)}{\bar{\Gamma}(B_s)} \equiv \frac{\Gamma(B_{s,short}) - \Gamma(B_{s,long})}{\bar{\Gamma}(B_s)} \simeq 0.18 \cdot \frac{f_{B_s}^2}{(200 \text{ MeV })^2} \cdot \qquad (24)$$

One can search for the existence of two different B_s lifetimes by comparing $\tau(B_s)$ as measured in $B_s \to \psi\phi$ and in $B_s \to l\nu X$. Analogously one can compare $\tau(B_d)$ as obtained from $B_d \to \psi K_S$, from $B_d \to \psi K^*$ and from $B_d \to l\nu X$. In the B_d case one theoretically expects a lifetime difference on the percent level only. Whether an effect of the size indicated in eq. (24) is large enough to be ever observed in a real experiment is of course a different matter. It has to be said, though, that eq.(24) does not represent a 'gold-plated' prediction. It is conceivable that the underlying computation underestimates the actual lifetime difference.

No detailed analysis has been performed yet on $\tau(\Lambda_b)$; simple estimates lead to the expectation

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} \sim 0.9. \qquad (25)$$

Present measurements yield [24]

$$\tau(B^+) = 1.66 \pm 0.11 \; psec \qquad (26a)$$

$$\tau(B_d) = 1.51 \pm 0.10 \; psec \qquad (26b)$$

$$\frac{\tau(B^+)}{\tau(B_d)} = 1.12 \pm 0.09 \qquad (26c)$$

$$\tau(B_s) = 1.54 \pm 0.24 \; psec \qquad (26d)$$

$$\tau(\Lambda_b) = 1.07 \pm 0.16 \; psec \qquad (26d)$$

While the predictions stated above on the lifetime ratios for B mesons, see eqs.(22,23), are quite consistent with the measurements, one cannot draw a definite conclusion at the moment. The Λ_b lifetime appears to be shorter than the B lifetimes, though:

$$\tau(B^+) - \tau(\Lambda_b) = 0.59 \pm 0.19 \; psec \qquad (27)$$

$$\tau(B_d) - \tau(\Lambda_b) = 0.44 \pm 0.19 \; psec \qquad (28)$$

Qualitatively this is expected although the size of the difference seems to be larger than anticipated.

3.2 Semileptonic Decays

The semileptonic branching ratio of beauty quarks depends on fundamental quantities of the Standard Model, namely KM parameters and quark masses. It is then the primary goal of the measurements to determine the numerical values of these quantities from the data. The theoretical challenge on the other hand consists in disentangling the effects of the strong interactions both in their perturbative as well as non-perturbative aspects.

The recent experimental studies have reached a new level of accuracy and reliability: a 'model-independent' ARGUS analysis yields [25]

$$BR_{SL}(B) = 9.6 \pm 0.5 \pm 0.4\% \qquad (29a)$$

whereas the CLEO collaboration finds [26]

$$BR_{SL}(B) = 10.65 \pm 0.05 \pm 0.33\% \qquad (29b)$$

using the model of Altarelli et al. for the *shape* of the lepton spectrum. It is an intrinsically phenomenological description, yet one should keep in mind that it provides a practically good approximation to the true QCD lepton spectrum as calculated through a $1/m_b$ expansion [6]. The present data thus clearly suggest:

$$BR_{SL}(B)|_{exp} \leq 11\% \qquad (30)$$

In a naive parton model where even perturbative QCD is ignored one obtains

$$BR_{SL}(b \to cl\nu) \simeq 15 \div 16\% \qquad (31)$$

i.e. a non-leptonic enhancement of $\sim 50\%$ has to be found to reproduce the data. At first sight this would not seem to represent a stiff challenge – yet so far we have failed to meet it!

There are non-perturbative as well as perturbative corrections to the semileptonic beauty branching ratio; our ignorance about the former was very considerable before the arrival of the Heavy Quark Expansion; we will discuss them in sequence.

3.2.1 Non-perturbative Corrections to BR_{SL}

The semileptonic and non-leptonic widths through order $1/m_b^2$ are given by [5, 27]:

$$\Gamma_{SL}(B) = \Gamma_0 \cdot \frac{\langle B|\bar{b}b|B\rangle}{2M_B} \cdot \left[I_0(x,0,0) + \frac{\mu_G^2}{m_b^2}(x\frac{d}{dx} - 2)I_0(x,0,0) \right], \qquad (32)$$

$$\Gamma_{NL}(B) = \Gamma_0 \cdot N_C \cdot \frac{\langle B|\bar{b}b|B\rangle}{2M_B} \cdot \left\{ A_0[\Sigma I_0(x) + \frac{\mu_G^2}{m_b^2}(x\frac{d}{dx} - 2)\Sigma I_0(x)] - \right.$$

$$\left. 8A_2\frac{\mu_G^2}{m_b^2} \cdot [I_2(x,0,0) + I_2(x,x,0)] \right\}. \qquad (33)$$

where the following notations have been used: I_0 and I_2 are phase-space factors:

$$I_0(x,0,0) = (1-x^2)(1-8x+x^2) - 12x^2\ln x, \ I_2(x,0,0) = (1-x)^3, \ x = (m_c/m_b)^2 \quad (34)$$

$$I_0(x,x,0) = v(1 - 14x - 2x^2 - 12x^3) + 24x^2(1-x^2)\ln\frac{1+v}{1-v}, \ v = \sqrt{1-4x} \quad (35a)$$

$$I_2(x,x,0) = v(1 + \frac{x}{2} + 3x^2) - 3x(1-2x^2)\ln\frac{1+v}{1-v} \qquad (35b)$$

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with $I_{0,2}(x,x,0)$ describing the $b \to c\bar{c}s$ transition, and $\Sigma I_0(x) \equiv I_0(x,0,0) + I_0(x,x,0)$; $A_0 = \eta J$, $A_2 = (c_+^2 - c_-^2)/6$ where $\eta = (c_-^2 + 2c_+^2)/3$, and J represents the effect of the subleading logarithms [28] (unknown for A_2). Moreover,

$$\mu_G^2 \equiv \frac{1}{2M_B} \langle B | \frac{1}{2} \bar{b} i\sigma \cdot Gb | B \rangle. \tag{36}$$

The matrix element $\langle B|\bar{b}b|B\rangle$ enters as an overall factor into both the semileptonic and non-leptonic width; its value does therefore not affect the semileptonic branching ratio. On the other hand $\langle B|\bar{b}i\sigma \cdot Gb|B\rangle$ which is determined from the observed $B^* - B$ mass splitting does, and it actually reduces $BR_{SL}(B)$ since $A_2 < 0$! Using

$$m_b^{pole} = 4.8 \text{ GeV} , \ m_c^{eff} \sim 1.35 \text{ GeV} \tag{37}$$

one finds

$$\delta BR_{SL}(B)|_{non-pert} \sim -0.02 \cdot BR_{SL}(B) \simeq -0.003 , \tag{38}$$

i.e. a very small reduction![3] Corrections of order $1/m_b^3$ have been analyzed in ref. [27]; as expected, they are estimated to be quite insignificant. This means that non-perturbative corrections can to a good first approximation be ignored in $BR_{SL}(B)$!

3.2.2 Perturbative Corrections to BR_{SL}

The preceding discussion shows that it is mainly the *perturbative* corrections that control the size of $BR_{SL}(B)$. They indeed generate a sizeable non-leptonic enhancement thus reducing $BR_{SL}(B)$, as desired; yet numerically they fall short of the goal. For one finds [27, 28]

$$BR_{SL}(B)|_{QCD} \geq 12.5 \ \% \tag{39}$$

It has been known for some time that the measured semileptonic branching ratios fall below the predicted ones. Now, however, one has reached a stage where one has to take such a deficit seriously since both the experimental and the theoretical analyses have become rather mature.

3.2.3 Fabula Docet?

An intriguing problem has arisen, which warrants serious consideration: how can one find an additional non-leptonic enhancement of at least 15 to 20 % to satisfy the bound of eq.(30)?

There are various scenarios for resolving this apparent puzzle:
(i) Improved data could move $BR_{SL}(B)$ above 12 % .
(ii) There could be a 'cocktail', i.e. a combination of several smallish effects all working in the same direction: the experimental number could inch up; higher order *perturbative* and non-perturbative corrections could turn out to be somewhat larger than estimated by us.

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[3]It should be noted that the corresponding effect is much larger in D decays – it leads to a roughly 50 % reduction in $BR_{SL}(D)$ and is essential for a self-consistent understanding of charm decays.

(iii) Non-perturbative corrections could conceivably be dramatically larger than anticipated by us. This certainly would require going beyond the 'Standard Version' of OPE; for it would presumably mean that even in beauty decays there are numerically significant non-perturbative corrections that enter through the coefficients in the OPE. In that scenario one would probably obtain considerably larger differences in the lifetimes of beauty hadrons than stated in eqs.(22, 25) above!

(iv) The most intriguing possibility would be the intervention of New Physics in B decays.

3.2.4 Lepton Spectra

The method outlined above has been extended to treat the lepton energy spectra in $H_b \to l\nu X$ transitions. The expansion is now in $1/(1-y)m_b$ rather than in $1/m_b$, where $y = E_l/E_l^{max}$ denotes the normalized lepton energy (for $b \to u$ decays $y = 2E_l/m_b$). This expansion is obviously and necessarily singular at $y = 1$, i.e. in the endpoint region, and one has to apply care in interpreting the results there.

To order $1/m_b^2$ the spectrum $d\Gamma/dy$ is evaluated without any free parameters with the non-perturbative corrections entering through the expectation values G_B and K_B; in practice there is at present some numerical uncertainty in the size of the kinetic energy term K_B, as already mentioned. The shape of the spectrum thus derived from QCD turns out to be remarkably similar to the one obtained from the phenomenological AC^2M^2 model [16] that has been fitted to the data. An important element of that model was the introduction of a Fermi motion ascribed to the heavy flavour quark. It has been found [6, 17, 13] that this notion of Fermi motion finds a natural home also in a rigorous QCD treatment: one can define a universal distribution function that describes the motion of the beauty quark inside H_b irrespective of the specifics of the decay process; to obtain the observable spectrum predicted from QCD one has to fold this distribution function with the primary lepton spectrum from beauty decays [4]. The properties of this distribution function control the shape of the lepton spectrum in the endpoint region. The second moment of the distribution function is given by $\langle \vec{p}^2 \rangle$. One can also show that this function as a matter of principle cannot be obtained from a non-relativistic hadronic wavefunction, although the b quark Fermi motion is of a non-relativistic nature [13].

Lastly, one finds that sizeable differences can arise in the endpoint spectrum of B_d vs. B^- mesons to order $1/m_b^3$ due to WA in the KM suppressed decays [42]. A detailed study of this difference in the spectra can provide information about the four-fermion matrix elements driving WA in nonleptonic decays and affecting PI.

3.3 Other Inclusive Non-Leptonic Decays

The $1/m_b$ expansion allows not only to calculate the overall non-leptonic and semileptonic rates, but also various sub-classes, like, e.g., those non-leptonic transitions that are driven by $b \to c\bar{c}s$, $b \to u\bar{u}d$ etc. While we have expressed some notes of caution

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[4]One should remember however that the universality holds only as long as the mass of the final state quark is the same. In QCD the corresponding functions that enter here are quite different for $b \to c$ and for $b \to u$ (or $b \to s + \gamma$) transitions.

about the prediction for $b \to c\bar{c}s$ reactions since the energy release there is not very large, no such caveat applies to the $b \to u\bar{u}d$ process: the expression in eq. (33) is easily adapted by the obvious substitutions: V_{ub} for V_{bc} and $m_u = 0$ for m_c; the expectation values of the local operators remain the same. Whether the prediction for the KM suppressed, inclusive non-leptonic B decays can ever be tested with any decent accuracy, is of course a different matter.

3.4 Radiative Beauty Decays

The non-perturbative contributions to $\Gamma(H_b \to s+\gamma)$ through order $1/m_b^2$ are obtained in a straightforward manner [29]:

$$\Gamma(H_b \to X_s + \gamma) = \Gamma(b \to s\gamma) \left(\frac{\langle H_b|\bar{b}b|H_b\rangle}{2M_{H_b}} - \frac{\langle H_b|\bar{b}i\sigma \cdot Gb|H_b\rangle}{2M_{H_b}m_b^2} + ... \right) \qquad (40)$$

where we have set $m_s = 0$. From eqs.(40) and (32) one reads off that the ratio $\Gamma(H_b \to s+\gamma)/\Gamma(H_b \to l\nu X)$ remains practically unaffected by the non-perturbative corrections.

More interesting effects arise in the photon spectrum: to lowest order it is given by a single line reflecting the two-body nature of $b \to s + \gamma$; both perturbative and non-perturbative corrections turn it into a continuous spectrum. Yet this will not be discussed here; the reader is referred to a quickly expanding literature [13, 33, 34] on that topic.

3.5 Charm Multiplicity in the Final State

Because of the KM hierarchy $|V(cb)|^2 \gg |V(ub)|^2$ and since the transition $b \to c\bar{c}s$ occurs one realizes that

$$N_{charm} \equiv \frac{\text{Number of charm states}}{B \text{ decay}} > 1 \qquad (41)$$

has to hold where charmonia states enter into the book keeping with a charm multiplicity of two. Since there is not enough phase-space for $b \to c\bar{u}d$, $cl\nu$ to transmogrify itself into $B \to D\bar{D}D$, $D\bar{D}Dl\nu$ etc., one knows without ado that each of these transitions will yield exactly one charm state per B decays. It used to be stated that one actually predicts $N_{charm} = 1.15$ because of $BR(b \to c\bar{c}s) = 0.15$. The prediction for the latter quantity depends of course quite sensitively on the values adopted for the ratio of the charm to the beauty quark mass, as expressed by the function $I_0(x, x, 0)$ defined in eq. (35a) with $x = (m_c/m_b)^2$. Using values for m_c and m_b as inferred from Heavy Quark Expansions one actually finds

$$N_{charm} \sim 1.2 \div 1.3 \qquad (42)$$

The data exhibit a considerably lower charm content, namely [35]

$$N_{charm} = 0.932 \pm 0.10 \quad ARGUS \qquad (43a)$$

$$N_{charm} = 1.026 \pm 0.057 \quad CLEO \qquad (43b)$$

Taking eqs.(42, 43) at face value one would have to state the existence of a significant 'charm deficit'. On the other hand one should keep in mind that in the transition $b \to c\bar{c}s$ the energy release is not that large. Thus the $1/m_b$ expansion has to be applied with a grain of salt here: the non-perturbative as well as perturbative corrections in $b \to c\bar{c}s$ could be larger than expected and negative. But one conclusion can be drawn in any event: attempting to lower the predicted semileptonic branching ratio for B mesons by increasing $BR(b \to c\bar{c}s)$ would fail!

3.6 Final State Interactions

Invoking strong final state interactions (FSI) – in the form of phase shifts and absorption – to escape conflict with the data represents a time-honoured and popular tool in phenomenological analyses; FSI are typically introduced in an ad-hoc fashion with an obscure dynamical foundation. Two attitudes towards FSI seem to prevail in the community: (a) It constitutes an *a priori* hopeless enterprise to account for FSI even in a semi-quantitative way because they are generated by strong re-scattering processes among real hadrons. (b) Whatever the origin of FSI, they are actually suppressed in heavy flavour decays.

Such a situation is quite unsatisfactory, not because FSI are by themselves enlightening – they certainly are not – but because they represent a *conditio sine qua non* for the observability of direct CP violation in B decays. Attitude (b) would suggest that direct CP asymmetries will be small in B decays; while attitude (a) on the other hand holds out the hope that one might find sizeable direct CP asymmetries, it would make their interpretation quite ambiguous.

The experience we have gained recently from the QCD treatments of heavy flavour decays leads us to the following expectation: FSI in *exclusive* decays may well possess a rather complex and non-trivial pattern that we cannot predict at present; yet re-scattering in *inclusive* transitions must be treatable in QCD using the same methodology that has been introduced in the previous sections.

One estimates FSI effects in non-leptonic $B \to X_{\text{charmless}}$ decays by analyzing the interference of the Penguin amplitude $b \to s[d]\bar{q}q$ with the tree-level KM suppressed $b \to u\bar{u}s[d]$ amplitude [30]. These two amplitudes possess a "hard" FSI phase difference δ_P due to the on-mass-shell intermediate c and $\bar{c}$ quarks in the Penguin process; one easily finds $\delta_P \sim 0.1$. It was shown [31] that all higher order gluon corrections can be summed up in a compact way; they reduce δ_P by just ~ 20 %. This demonstrates that the perturbative expansion makes sense also for this quantity and thus refutes earlier claims to the contrary. An elegant analysis can be given [31] that involves rather general arguments based on gauge invariance and the equation of motion in QCD, and shows the strong suppression of the decays of the type $b \to s + gg[ggg]$; the latter can actually be traced back to the first genuine QCD study of strange decays [32].

Only the purely perturbative corrections to FSI have been addressed in ref. [31]. Non-perturbative contributions still await a detailed analysis; on general grounds one expects them to be sizeable, yet at the same time theoretically tractable!

3.7 B **Decays into Charmonia**

A very intriguing class of reactions is provided by the decays into charmonia states: $B \to \psi/\psi'/\chi + X$. No rigorous QCD analysis of these modes has been given yet, but reasonable phenomenological treatments do exist. Folding the wavefunctions for colour singlet $\bar{c}c$ boundstates with the $\bar{c}c$ spectrum in $b \to c\bar{c}s$ one can make predictions about the relative as well as the absolute rates [36]:

$$\Gamma(b \to \psi_{dir}X) : \Gamma(b \to \psi'X) : \Gamma(b \to \eta_c X) : \Gamma(b \to \chi_1 X) \simeq$$

$$\simeq 1 : 0.31 : 0.57 : 0.27 \tag{44a}$$

Furthermore

$$\Gamma(b \to \chi_2 X) = 0 \tag{44b}$$

holds in this ansatz. These ratios depend of course on the wave functions chosen, but are independent of c_1, c_2 and N_C. The absolute widths on the other hand depend on these parameters

$$\Gamma(b \to \psi_{dir}X) \simeq (0.42 \div 0.45)(c_2 + \frac{1}{N_C}c_1)^2\Gamma_0 \tag{45}$$

while being rather insensitive to the values adopted for the quark masses. For the part of the ψ wave function that is relevant here is probed also in $\psi \to e^+e^-$ and can therefore be obtained from the data on that electromagnetic decay. In eq.(45) the width for *directly* produced ψ was given; including the feed-down from $b \to \psi'/\chi_1+X$ one finds for the total rate for ψ production:

$$\Gamma(b \to \psi + X) \simeq (0.52 \div 0.56)(c_2 + \frac{1}{N_C}c_1)^2\Gamma_0 \tag{46}$$

and, therefore,

$$BR(B \to \psi + X) \simeq (0.27 \div 0.4)\,\%. \tag{47a}$$

If the $1/N_C$ term in eq. (46) is omitted one gets, instead,

$$BR(B \to \psi + X) \simeq (0.83 \div 1.2)\,\%. \tag{47b}$$

Comparing these predictions with the data from ARGUS and CLEO

$$BR(B \to \psi X) = (1.09 \pm 0.04 \pm 0.07)\% \tag{48a}$$

$$BR(B \to \psi' X) = (0.30 \pm 0.05 \pm 0.03)\% \tag{48b}$$

$$BR(B \to \chi_1 X) = (0.54 \pm 0.15 \pm 0.14)\% \tag{48c}$$

one draws three conclusions:
• The prediction on $BR(B \to \psi'X)$ vs. $BR(B \to \psi X)$ is in good agreement with the data.
• For the natural value $1/N_C = 1/3$ the improved parton model expression in eq.(46) yields a branching ratio that is too low by a factor of $\sim 3 - 4$. With $1/N_C = 0$ the observed branching ratio is reproduced, as it is for $1/N_C \simeq 0.45$! This means – not

surprisingly – that for $b \to c\bar{c}s$ transitions the deviations from this model are large and not fully implemented through employing a hadronic wavefunction. It remains a challenge to theory to identify those corrections.

• The prediction on $BR(B \to \chi_1 X)$ vs. $BR(B \to \psi X)$ is $\sim 2\,\sigma$ low. This could be blamed on a less than optimal choice for the χ_1 wavefunction. As already mentioned the relevant part of the ψ and ψ' wavefunctions is directly calibrated by the data on $\psi, \psi' \to e^+ e^-$; for the χ_1 wavefunction such a cross check does not (yet) exist and one has to rely on a specific ansatz for the interquark potential, which introduces additional systematic uncertainties. There exists of course another possibility [37]: the $c\bar{c}$ pair could be produced in a colour *octet* configuration which then sheds its colour charge through emission of a soft gluon and transforms itself into a colour singlet P wave charmonium state. Naive factorization does not hold anymore in the presence of this second source of χ production in B decays; its weight depends on the probability for a colour octet $c\bar{c}$ to transmogrify itself into a P-wave charmonium state which cannot be predicted from first principles at present. On the other hand one can extract its size from the requirement to saturate the observed $B \to \chi_1 X$ rate; subsequently one can then predict the rate for $B \to \chi_2 X$. One finds

$$\Gamma(B \to \chi_1 X) \sim \Gamma(B \to \chi_2 X)$$

in contrast to eq. (44).

4   Exclusive Two-Body Decays of Beauty

A discussion of exclusive non-leptonic decays has to be opened with a note of caution:

First Theoretical Caveat: The relationship between inclusive and exclusive transition rates is nothing short of delicate!

This piece of common sense can be illustrated by the following example [4]. Consider the *corrections* to the decay width of a $(Q\bar{q})$ meson that are induced by Weak Annihilation, see Fig. 2

As indicated there are three cuts in the $Q\bar{q} \to Q\bar{q}$ forward scattering amplitude representing different final states, one with an on-shell gluon and the other two with a (slightly) off-shell gluon in the form of a $\bar{q}q$ pair. Summing over these three cuts yields an overall correction $|T|^2$ that remains finite even in the limit $m_q/m_Q \to 0$ [4, 11]. However a very striking pattern emerges when one considers separately the three 'exclusive' channels (a), (b) and (c) [5]: The contribution (b) constituting the square of an amplitude is positive; in the limit $m_q/m_Q \to 0$ it is dominated by a term $+(m_Q^2/m_q^2)|T|^2$ with T denoting a quantity that is regular in the limit $m_q/m_Q \to 0$; the contributions (a) and (c) on the other hand represent interference terms that, taken together, are of the form $-(m_Q^2/m_q^2)|T|^2$ for $m_q/m_Q \to 0$ so that the sum of (a), (b) and (c) possesses a regular limit. We want to draw the following lesson from this discussion: a small effect in an overall rate can be due to large cancellations among subclasses of decays. The relevance of this statement will become clearer later on.

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[5]In the real world these three channels can of course not be distinguished; yet this academic model can illustrate the relevant point.

4.1 Phenomenological Models

A popular phenomenological model of non-leptonic two-body decays of charm and beauty was suggested in ref. [39]. There are three main ingredients in all models of this type:

(i) One assumes factorization, i.e. one uses $\langle M_1 M_2 | J_\mu J_\mu | D \rangle \simeq \langle M_1 | J_\mu | D \rangle \cdot \langle M_2 | J_\mu | 0 \rangle$ to describe $D \to M_1 M_2$. (ii) One employs one's favourite hadronic wavefunctions to compute $\langle M_1 | J_\mu | D \rangle$. Very recently Heavy Quark Symmetry and Chiral Symmetry (for the light quarks) have been incorporated into these wavefunctions [40]. (iii) All two-body modes are then expressed in terms of two free fit parameters $a_1^{(c)}$ and $a_2^{(c)}$, with $a_1^{(c)}$ controlling the 'class I' $D^0 \to M_1^+ M_2^-$ and $a_2^{(c)}$ the 'class II' $D^0 \to M_1^0 M_2^0$ transitions; both quantities contribute coherently to the 'class III' transitions $D^+ \to M_1^0 M_2^+$. The analogous procedure is applied to B decays allowing though for $a_1^{(b)} \neq a_1^{(c)}$ and $a_2^{(b)} \neq a_2^{(c)}$.

With these two free parameters $a_{1,2}^{(c)}$ (and some considerable degree of poetic license in invoking strong final state interactions) one obtains a decent fit for the D^0 and D^+ modes (much less so, however, for D_s decays). The situation is rather similar in B decays. One has to point out, though, that this success is helped by a considerable ambiguity in the estimates of the matrix elements of the type $\langle \pi | J_\mu | B \rangle$ and by the forgiving imprecision in many of the branching ratios measured so far.

Exclusive decay rates depend sensitively on long-distance dynamics. In the factorization approximation large distances enter through simple hadronic matrix elements $\langle M | J_\mu | M \rangle$ and $\langle M | J_\mu | 0 \rangle$. The coefficients a_1 and a_2 are:

$$a_1 = c_1 + \xi c_2 \tag{49a}$$

$$a_2 = c_2 + \xi c_1 \tag{49b}$$

where c_1 and c_2, see eq. (17), include the radiative QCD corrections due to hard gluons; the quantity ξ is introduced as a fudge factor. Literally speaking, factorization implies $\xi = 1/N_C = 1/3$. Quite often ξ is treated as a fit parameter reflecting our ignorance of how a quark and an anti-quark that are *not* correlated in colour combine to form a meson. Thus, deviations of ξ from 1/3 parametrize, in a certain way, non-factorizable contributions. The non-factorizable contributions show up even in perturbation theory, but these seem to be numerically small. An example of the non-perturbative contribution which can violate factorization is provided by the strong final state interactions (FSI). The latter can easily change the value of ξ. Some prominent features of FSI are actually added in an ad-hoc fashion; yet even so one would a priori *not* expect that all these non-perturbative corrections can by and large be lumped into a single fudge factor ξ. At this point we would like to express another note of caution:

Second Theoretical Caveat: There exists no general proof of the Dogma of Factorization for real hadrons; we actually consider it unlikely to be of universal validity, in particular for class II and III transitions. It thus makes eminent sense to subject this dogma to as many different experimental tests as possible.

A fit to eleven exclusive hadronic B branching ratios yields [41]

$$a_1 = 1.05 \pm 0.03 \pm 0.10 \tag{50a}$$

$$a_2 = 0.227 \pm 0.012 \pm 0.022 \qquad (50b)$$

A few comments are in order here:

(i) The results stated in eqs. (50) are bad news for a popular program to infer a_1 and a_2 from perturbative QCD plus factorization: the effective transition operators are renormalized with coefficients c_+, c_- generated from perturbative QCD; then the hadronic matrix elements are factorized. The coefficient a_2 does not come out correctly. Moreover, the so-called rule of $1/N_C$ (this is an *ad hoc* assumption that only the terms leading in $1/N_C$ are to be retained), which miraculously helped in the two-body decays of charm, seems only to worsen the situation for beauty. The program "factorization plus the rule of $1/N_C$" implies $a_1 = c_1$, $a_2 = c_2$, in clear conflict with the findings in B decays, eqs. (50), since c_2 is negative! Yet even before the following statement applied:

Third Theoretical Caveat: It is very unlikely that the rule of retaining only leading terms in $1/N_C$ is universally implemented in QCD. Our analysis of inclusive heavy-flavour decays actually found cases where this rule was dynamically realized (a) for D as well as for B decays, (b) for D, but not for B decays, or (c) for neither [4, 5, 43] (see also Sect. 4.2).

(ii) The most striking feature of eqs. (50) is that the relative sign between a_1 and a_2 is positive, i.e. that a *constructive* interference occurs in class III transitions, i.e. in exclusive two-body B^- decays! This is surprising in four aspects: (a) It is in clear contrast to the situation in D decays where the D^+ modes suffer from a destructive interference. (b) While it does not pose any fundamental problem for the BSW model, it represents a basic failing for the $1/N_C$ ansatz which predicts a destructive interference in the two-body modes of B as well as of D decays. (c) The observed enhancement of the B^+ rates is remarkably uniform [41]:

$$BR(B^- \to D^0 \pi^-)/BR(B^0 \to D^+ \pi^-) = 1.71 \pm 0.38 \qquad (51a)$$

$$BR(B^- \to D^0 \rho^-)/BR(B^0 \to D^+ \rho^-) = 1.60 \pm 0.46 \qquad (51b)$$

$$BR(B^- \to D^{*0} \pi^-)/BR(B^0 \to D^{*+} \pi^-) = 1.79 \pm 0.39 \qquad (51c)$$

(d) It raises the question of whether the same constructive interference might occur for the *inclusive* rate thus shortening $\tau(B^-)$ relative to $\tau(B_d)$ rather than lengthening it.

As stated in Sect. 3 the B^- lifetime is predicted to exceed the B_d lifetime, albeit by a few percent only; yet one has to keep in mind the *First Theoretical Caveat* stated in the beginning of this section: a small correction in the inclusive rate is quite likely to be made up by large contributions of alternating signs coming from different classes of exclusive transitions.

For a better understanding of these problems one has to progress to treatments that are rooted more firmly in QCD. However we would like to first stress the important lessons we have learnt and are still learning from these phenomenological descriptions:

– They have yielded quite a few successful 'predictions' in a user-friendly way providing a useful pattern for cataloguing dozens of the decay rates, a reference frame for more sophisticated approaches.

– They have helped us considerably in focusing on the underlying theoretical problems such as the question of factorization or the $1/N_C$ rule.

– From their fits to the data they provide us with valuable, albeit indirect information on the final state interactions. Such information is crucial in studies of direct CP violation.

4.2 Theoretical Analysis

The first treatment of two-body decays of heavy flavour hadrons that is intrinsically connected to QCD was given in ref. [43] some time ago for D decays, within the framework of the QCD sum rules. To extend this analysis to non-leptonic B decays is a rather non-trivial undertaking; work on treating $B \to J/\psi K$ through QCD is in progress now [44].

Recently there has been progress in analysing *deviations* from the $1/N_C$ rule (see Sect. 4.1) in some exclusive two-particle B decays [45]. It has been shown how non-perturbative effects in QCD can provide a dynamical realization of this rule in some decay channels, but not in others. The key role is played by the same chromomagnetic operator $\sigma \cdot G$ that was repeatedly discussed above in connection with the inclusive decays. It generates non-perturbative corrections that are specific for the decay channel considered, yet can be estimated in a model-independant way.

The general method can be illustrated through the example of the decay $B^0 \to D^+\pi^-$. The non-factorizable terms in this decay are reducible to the amplitude $\langle D\pi|\mathcal{L}_{oct}|B\rangle$ where

$$\mathcal{L}_{oct} \sim (\bar{c}\gamma_\mu(1-\gamma_5)t^a b)(\bar{d}\gamma_\mu(1-\gamma_5)t^a u),$$

with t^a denoting the colour matrices. Colour has to be exchanged between the brackets because otherwise the light quarks can not form the pion; this can be done by a soft gluon.

To estimate this effect one considers the time ordered correlation function

$$\mathcal{A}^\beta \equiv \int d^4x e^{iqx} \langle D|T\{\mathcal{L}_{oct}(x), A^\beta\}|\bar{B}\rangle \tag{52}$$

where an auxiliary axial current $A^\beta = \bar{u}\gamma^\beta\gamma_5 d$ annihilates the pion and q is an external momentum flowing through A^β. To calculate the correlator one adopts a similar procedure as in QCD sum rules. After continuing $\mathcal{A}^\beta$ into the Euclidean region $-q^2 = Q^2 \sim 1$ GeV ^{2} one invokes duality in the following way: on the one hand one expresses $\mathcal{A}^\beta$ in terms of $M_{non-fact}$, the non-factorizable part of the amplitude, as obtained from $\mathcal{L}_{oct}$:

$$\mathcal{A}^{oct}(Q^2) = M_{non-fact}\frac{f_\pi q^\beta}{q^2} + ... \tag{53}$$

where +... denotes the contributions from the higher resonances produced by the axial current A^β; on the other hand one applies an OPE to $\mathcal{A}^\beta$ to find

$$\mathcal{A}^{oct}(Q^2) = i\frac{1}{4\pi^2}\frac{q_\alpha q_\beta}{q^2}\langle D|\bar{c}\gamma_\mu\gamma_5\tilde{G}_{\alpha\mu}b|B\rangle + ... \tag{54}$$

where +... now denotes pre-asymptotic corrections from higher dimensional operators. If both masses, m_c and m_b, are treated as heavy, the matrix element in eq. (54) is related to the expectation value of the chromomagnetic operator σG and is expressed in terms of μ_G^2, see eq. (36b). Comparing the leading terms in eqs. (53, 54) one obtains for the ratio of the non-factorizable to the $1/N_C$ factorizable parts of the amplitude

$$r_{non-fact} = -\frac{N_C \mu_G^2}{4\pi^2 f_\pi^2} \tag{55}$$

An essential difference to the usual QCD sum rules analysis is worth noting: rather than the vacuum condensates we deal here with hadronic expectation values between heavy flavour hadron states. Numerically one finds

$$r_{non-fact}(B^0 \to D^+\pi^-) \sim -1 \tag{56}$$

and likewise for the mode $B^0 \to D^+\rho^-$; i.e. in these two modes non-factorizable contributions which are necessarily of order $1/N_C$ basically cancel against the non-leading factorizable contributions of order $1/N_C$ thus leading to a dynamical realization of the $1/N_C$ rule. The findings are similar for $B \to D\bar{D}$ decays. Yet the situation changes for other modes: in $B \to D^*\bar{D}^*$ the non-factorizable contributions are suppressed; likewise in $B^0 \to D^{*+}\pi^-$, $D^{*+}\rho^-$.

There are two general conclusions we want to draw from this analysis: (i) The weight of non-factorizable contributions is indeed quite channel dependent – as expected. (ii) The discussion so far was given for class I transitions where we found that the coefficient a_1 is not quite universal, but has some channel dependence. The situation is much more complex and actually at present unclear for class II (and III) transitions. For the a_2 amplitude contains the matrix element $\langle B|\bar{b}\gamma_\nu g\tilde{G}_{\alpha\nu}u|\pi\rangle$. This formfactor, unfortunately, can not be determined from HQET.

This treatment has reached so far only the qualitative or at best semi-quantitative stage. No detailed analysis has yet been given about the question whether one can really suppress the contributions from higher resonances and from higher dimensional operators in the sum rule of eqs.(53, 54) to a sufficient degree. Furthermore, the operator product expansion, eq. (54), assumes that $(M_B - M_D)/(M_B + M_D) << 1$; in the real world this parameter is rather of order unity. Also radiative corrections have not been included yet. Further details can be found in ref. [45]. Thus, much more theoretical work is needed before such a treatment finds its definite form.

4.3   Prizes to be Attained

The theoretical methods one applies to exclusive decays are often not of the most lucid kind. Yet they are essential (if imperfect) tools for addressing fundamental questions. Let us cite just one topical example:

$\Delta\Gamma(B_s)$, i.e. the lifetime difference between $B_{s,short}$ and $B_{s,long}$, is usually computed from the quark box diagram with internal c (and u) quarks, leading to a result like the one quoted in eq. (15). However the weight of such a short-distance contribution to $\Delta\Gamma(B_s)$ is much more uncertain than that of the local contribution to $\Delta m(B_s)$, which is given by virtual top exchanges. It is therefore conceivable in principle that

nontrivial long-distance dynamics could provide in reality a larger contribution. This possibility has been analysed by the Orsay group [46] considering separate transitions

$$B_s \to D_s^{(*)} \bar{D}_s^{(*)}, \psi\phi, \psi\eta \to \bar{B}_s$$

and it has been inferred from present data $\Delta\Gamma/\bar{\Gamma}(B_s) \simeq 0.15$ in satisfactory agreement with what one finds using the quark box diagram, eq. (24).

5 Summary and Outlook

5.1 Status

New and more powerful second-generation theoretical technologies are emerging: QCD sum rules, Heavy Quark Symmetry, $1/m_Q$ expansions and lattice simulations of QCD. They are leading to

• significant conceptual progress, namely a better understanding of (i) the form and size of non-perturbative corrections, (ii) the relationship between charm and beauty decays, where the former play the role of a microscope for the non-perturbative corrections in the latter, and (iii) the differences and similarities of baryon vs. meson decays;

• the realization that charm and beauty baryons deserve detailed studies in their own right.

• a quantitative phenomenology that is genuinely based on QCD:

$$\frac{\tau(D^+)}{\tau(D^0)} \sim 2; \quad BR_{SL}(D^+) \sim 16\%, \quad BR_{SL}(D^0) \sim 8\%;$$

$$\frac{\tau(D_s)}{\tau(D^0)} \sim 1.0 \pm \text{ a few per cent}, \tag{57}$$

i.e. the data are reproduced within the accuracy of the expansion.

$$\frac{\tau(B^-)}{\tau(B_d)} \simeq 1 + 0.05 \cdot \frac{f_B^2}{(200 \text{ MeV })^2}; \quad BR_{SL}(B) \geq 12\% \tag{58}$$

$$\frac{\Delta\Gamma(B_s)}{\bar{\Gamma}(B_s)} \simeq 0.18 \cdot \frac{f_{B_s}^2}{(200 \text{ MeV })^2}, \quad \frac{\tau(\Lambda_b)}{\tau(B_d)} \sim 0.9 \tag{59}$$

$$d\Gamma(B^- \to l\nu X)/dE_l \neq d\Gamma(B_d \to l\nu X)/dE_l \tag{60}$$

The prediction for $BR_{SL}(B)$ is somewhat larger than present CLEO and ARGUS measurements. This could turn out to be a serious – or intriguing – discrepancy. It could conceivably signal the presence of anomalously large higher-order contributions that so far have not been included in the theoretical expression. In that case one would expect lifetime ratios for beauty hadrons to differ more from unity than stated in eqs. (58) and (59) [27].

Phenomenological models for nonleptonic two-body modes are encountering discrepancies with more precise data; yet they continue to be useful and help us in focusing on the underlying theoretical issues.

5.2 Future

One can expect a refinement of and increased cooperation (rather than just coexistence) between the second-generation theoretical technologies. On the experimental side one can hope for

– lifetime measurements for individual beauty hadrons with 10% accuracy soon and percent accuracy in the longer run;

– data on $\tau(B_s)$ *separately* from $B_s \to \psi\phi$ and from $B_s \to l\nu D_s$;

– perform the 'class I, II, III' phenomenology *individually* for *KM* allowed and *KM* suppressed B decays.

The primary goal in all these efforts is to be able to exploit to the fullest over the next 20 years or so, the discovery potential or even discovery guarantee that awaits us in beauty physics. It certainly would be a crime not to make all conceivable efforts to obtain the required experimental facilities.

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FIGURE CAPTIONS

Fig. 1

Corrections to the $b\bar{q} \to b\bar{q}$ forward scattering amplitude induced by Weak Annihilation. The three cuts (a), (b) and (c) represent different final states for the $b\bar{q}$ decay.

Fig. 2

(a) Diagram generating the operator $\bar{b}b$.
(b) Diagram describing Weak Annihilation.
(c) Diagram describing Pauli Interference.